Turing instability leads oscillatory systems to spatiotemporal chaos

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Abstract: We present that the Turing instability can lead the uniform state to a newly identified chemical spatiotemporal chaos instead of spatially periodic steady states. (振動性の反応拡散系では、Turing 不安定性が、空間周期構造ではなく新しいタイプの時空カオスを生じさせることがある。)

The spontaneous formation of chemical spatial structure from an initially uniform state was predicted in 1952 by Turing and observed in the 1990's in controlled laboratory experiments. The Turing (T) mechanism is now widely accepted and discussed in diverse disciplines. The spontaneous formation of temporal structure as well as that of spatial structure is an important subject in the study of dissipative systems: oscillation, synchronization, etc. The purpose of this paper is to consider what structure is exhibited by oscillatory systems immediately above the T instability. In oscillatory systems, a uniform mode called phase mode is neutrally stable. This is a result of the spontaneous breaking of time translational symmetry. In the sideband of the phase mode, there are long-wavelength modes that decay or grow much slowly. In the latter case, the systems exhibits the Kuramoto-Sivashinsky spatiotemporal chaos. However, in this paper, we concentrate the former case, i.e., we consider Benjamin-Feir (BF) stable regime. Fig. 1 shows the dispersion curve immediately above the T criticality in the BF stable regime. The long-wavelength modes decay much slowly when standing without nonlinear interaction

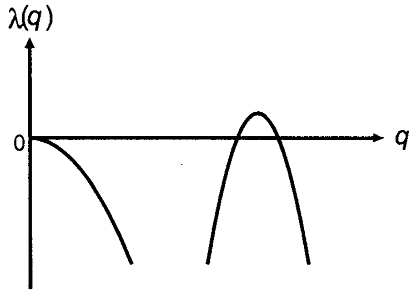

Figure 1: Stability eigenvalue as a function of the perturbation wavenumber for the uniform oscillating solution in BF stable and T unstable regime.

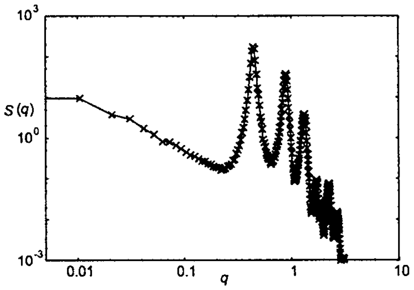

Figure 2: Spatial power spectrum of $\partial_r \arg(X_2/X_1)$.

with the other modes. Its characteristic time scale is comparable to that of the T modes, if the

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systems are in the neighborhood of the T criticality. Thus, these two bands of modes, weakly stable long-wavelength modes and slightly unstable short-wavelength modes, can interact in general. This interaction can lead the initially uniform state to spatiotemporal chaos. Similar onset of spatiotemporal chaos was discovered by Tribelsky *et al.* for the equation,

$$\partial_t \phi = -\partial_r^2[\epsilon - (1+\partial_r^2)^2]\phi - (\partial_r \phi)^2, \tag{1}$$

in which the linear stability of $\phi = 0$ is qualitatively the same as Fig. 1. This equation was originally proposed by Nikolaevskii to describe the propagation of longitudinal seismic waves in viscoelastic media. We show that this equation can be derived from a certain class of oscillatory reaction-diffusion (RD) systems in the neighborhood of a codimension-two T-BF point [1]. The numerical results support our argument and show robustness of this type of spatiotemporal chaos. We derive critical exponents of chaotic fluctuations [2] and study bifurcation scenario to this chaos. Fig. 2 shows the spatial power spectrum for a simple RD model belonging to the above-stated class of RD systems,

$$\begin{aligned} \partial_t X_1 &= \mu X_1 - \alpha X_2 - (X_1 - \beta X_2)(X_1^2 + X_2^2) + k_1 S + \delta \partial_r^2 X_1 \\ \partial_t X_2 &= \mu X_2 + \alpha X_1 - (X_2 + \beta X_1)(X_1^2 + X_2^2) + k_2 S + \delta \partial_r^2 X_2 \\ \tau \partial_t S &= X_1 - S + D\partial_r^2 S, \end{aligned}$$

where $\mu = 0.1$, $\alpha = 5$, $\beta = 1$, $\delta = 0.05$, $k_1 = 0.2908$, $k_2 = -0.0608$, $\tau = 1.2$, and $D = 10$. These values of parameters realize the T-unstable regime close to the BF criticality. The initial configuration is the uniform oscillating solution $(X_1, X_2, S) = (0.281080, 0.141204, 0.031221)$ with a slight perturbation. The spectrum is found to monotonically decrease in the sideband of the phase mode and to have characteristic peaks at the T wavenumber and its harmonics. This is qualitatively quite similar to the spatial power spectrum of Eq. (1).

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References

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